

A NEW APPROACH TO BROADBAND SAW DIFFRACTION THEORY

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ABSTRACT

This paper describes an innovative technique for determining a two dimensional impulse response diffraction model from the wave equation using an impulse driver. The approach presented is unique in that it determines the point-to-point impulse response for the isotropic case using the basic wave equation of motion, and utilizes a double integral reduction technique to determine the tap-to-tap response. This takes into account diffraction effects, and provides an impulse response which is valid for the broadband case. Results of the analysis are presented for both the isotropic and anisotropic case.

INTRODUCTION

Many SAW device design techniques start with the basic assumption that the wavefront generated by an overlapping pair of electrodes layered on a piezoelectric crystal is uniform in the transverse direction. However, diffraction causes the wavefront to be distorted from this ideal shape, thereby introducing errors into the analysis. It is desirable to be able to compensate for diffraction effects when designing SAW devices. Several types of diffraction analyses have previously been developed for this purpose [2,4,6-10]. Most of these approaches, however, were based on traditional theories of diffraction, utilizing the Fresnel integral or Angular Spectrum of Plane Waves approach. The Fresnel integral, originally used to describe diffraction in optics, results in a narrowband approximation to the impulse response, while the Angular Spectrum of Plane Waves Approach requires a time consuming integration over frequency if broadband diffraction effects are to be analyzed. An asymptotic expansion has been previously presented by Tan and Flory [9] to reduce the complexity of these calculations.

The goal of this paper is to present the development of a model for the true impulse response of a SAW device by reconsidering the diffraction theory involved. An impulse driver is applied to the two dimensional wave equation, and a point-to-point impulse response is obtained. This approach does not limit the applicable bandwidth of the response, and eliminates the need to integrate over frequency, thus reducing calculational complexity. It should be noted that the use of the two dimensional wave equation to model the propagation of a surface acoustic wave, a three dimensional phenomenon, is certainly not

rigorous. It can be shown, however, that any errors introduced by the use of this equation can either be compensated for or are negligible. The model is first applied to isotropic substrates, then extended to the anisotropic case.

THEORY

Derivation of the Point Source Impulse Response

In developing and solving the two-dimensional wave equation, the surface wave amplitude at any position and time can be represented by a scalar function, which can be interpreted as a component of surface displacement, or electric surface potential [5]. Displacement will be used in this development.

The equation of motion of an isotropic, elastic substrate, including an impulse driving function and assuming no variation of displacement with z , reduces in cylindrical coordinates in two dimensions to the following

$$\left\{ \frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r,t)}{\partial r} - \frac{1}{c^2} \frac{\partial^2 u(r,t)}{\partial t^2} \right\} = - \frac{\delta(r)\delta(t)}{2\pi r} \quad (14)$$

where c is the surface wave velocity and $u(r,t)$ represents the component of displacement in question.

Taking the Hankel transform of order zero with respect to r of this equation yields the following differential equation,

$$\frac{\partial^2 U(\rho,t)}{\partial t^2} + \rho^2 c^2 U(\rho,t) = c^2 \frac{\delta(t)}{2\pi} \quad (15)$$

Solving this differential equation subject to initial conditions of zero velocity and displacement at time 0^- and the requirement that the response remain finite as $\rho \rightarrow \infty$ yields

$$U(\rho,t) = \frac{c \sin(c\rho t)h(t)}{2\pi\rho} \quad (3)$$

where $h(t)$ is the unit step function.

Taking the inverse Hankel transform yields the point to point time domain impulse response

$$u(r,t) = \frac{c h(ct-r)}{2\pi [(ct)^2 - r^2]^{1/2}} \quad (4)$$

The Fourier transform of this response yields the result

$$U(r, \omega) = \frac{1}{2} \chi_0 \left(\frac{j\omega r}{c} \right) \quad (5)$$

where χ_0 is the Bessel function of the second kind. Since the Bessel function is difficult to evaluate for use in a practical diffraction analysis, it is useful to consider approximations to this result. Approximating the time domain response by

$$u'(r, t) = \frac{c h(ct-r)}{2\pi [2r(ct-r)]^{1/2}} \quad (6)$$

(for $ct-r=2r$), the approximate frequency response is given by the Fourier transform of equation (6)

$$U'(r, \omega) = \frac{1}{4\pi} \left[\frac{2\pi c}{j\omega r} \right]^{1/2} e^{-j\omega r/c} \quad (7)$$

This result can also be obtained using an asymptotic approximation for the Bessel function of the second kind, and for most practical transducer geometries and substrates is indistinguishable from the exact transform expression for frequencies above a few megahertz. Since this approximation to the ideal point-to-point impulse response is valid in the high frequency region of interest, it can be used to determine the response at any frequency at a point due to an impulse applied to a tap.

Tap to Point Response

Using the principle of superposition, the response at any given observation point to an impulse applied to a tap can be found by integrating the point-to-point impulse response along the tap. This yields the following response in the time domain

$$h_{t-p}(y_p, t) = \int_0^L \frac{h(ct-r)}{2\pi [(ct)^2 - r^2]^{1/2}} dy \quad (8)$$

where y is the position along the transmitting tap, y_p is the position of the observation point

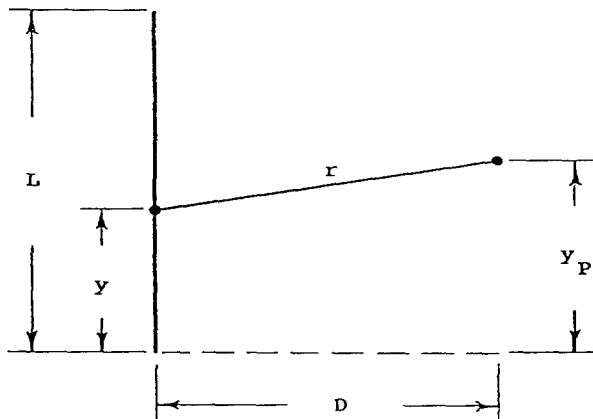
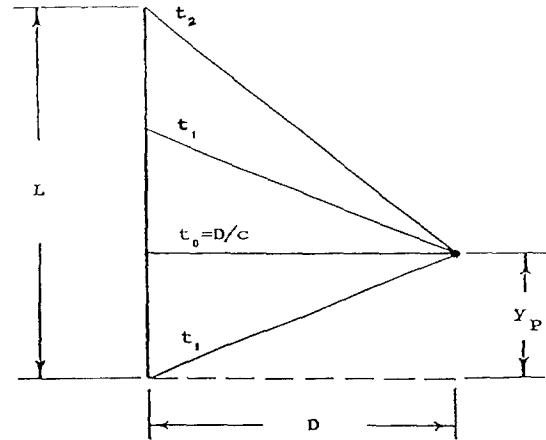


FIGURE 1

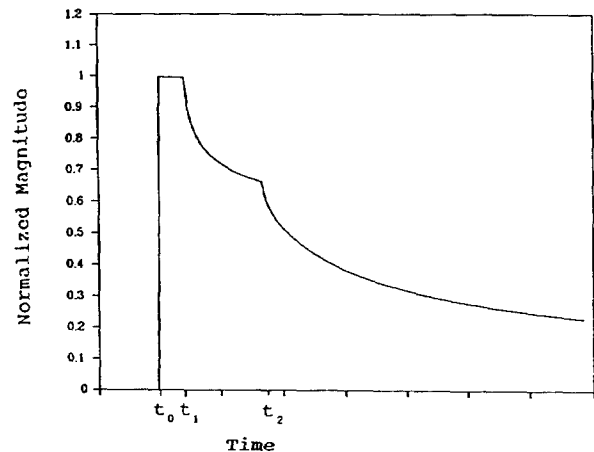
Coordinate System for Tap to Point Integration

relative to a line parallel to the bottom of the transmitting tap, and L is the tap length as shown in Figure 1.

This integration yields a response at point y_p such as the one shown in Figure 2. From Fig. 2(a), it can be seen that no response is expected at the point y_p until a time $t_0=D/c$ has been reached. From time t_0 to t_1 , there are two regions of the transmitting tap making equivalent contributions to the response at point y_p , while from t_1 to t_2 only one region contributes directly, and for times beyond t_2 there are no new contributions from the tap to the response at y_p . The behavior of the impulse response shown in Fig. 2(b) reflects these time limits. Prior to time t_0 , no response is detected at y_p . From t_0 to t_1 , the response is fairly constant while it drops off from t_1 to t_2 , the time frame when only one direct contribution is being made. For times beyond t_2 ,



(a) Propagation times for tap to point impulse response



(b) Tap to point impulse response

FIGURE 2

the response falls off towards zero, as required by the form of the solution to the two dimensional wave equation.

In the frequency domain, the tap to point integral becomes

$$H_{t-p} (y_p, \omega) = \int_0^L \frac{1}{4\pi} \left[\frac{2\pi c}{j\omega r} \right]^{1/2} e^{-j\omega r/c} dy \quad (9)$$

where the approximate frequency response, given in equation (7) has been used. Equation (9) is evaluated numerically and used to generate the transverse intensity or "beam" profiles at various distances from the source tap.

Tap to Tap Response

In order to determine the impulse response of a pair of parallel taps of arbitrary lengths and positions relative to one another, a double integral over the taps must be performed. This integral can be expressed in the time domain as follows

$$\int_{y_T=-b}^{L-a} \int_{y_R=a}^b \frac{h(ct-r)}{2\pi [(ct)^2 - r^2]^{1/2}} dy_R dy_T \quad (10)$$

or in the frequency domain as

$$\int_{y_T=-b}^{L-a} \int_{y_R=a}^b \left[\frac{2\pi c}{j\omega r} \right]^{1/2} e^{-j\omega r/c} dy_R dy_T \quad (11)$$

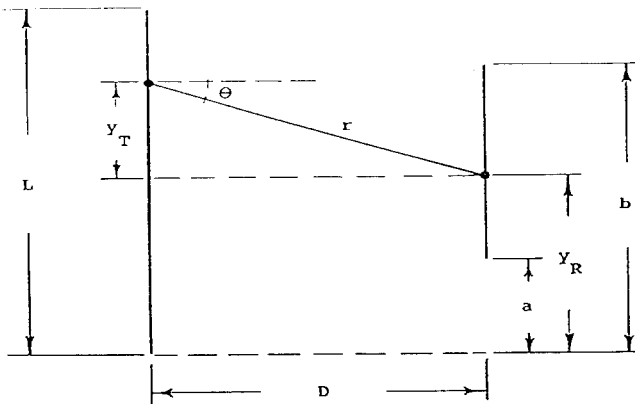


FIGURE 3

Coordinate System for Tap to Tap Double Integral

where y_R is the coordinate along the 'receiving' tap, y_T is the coordinate along the 'transmitting' tap relative to a horizontal line through y_R , and a, b, L, D , and r are as shown in Figure 3. The receiving tap is shown as shorter than the transmitting one for convenience, but by reciprocity the response of a short tap transmitting to a longer tap would be identical.

In evaluating this double integral, a technique called double integral reduction, as presented by Datta [2], is used to reduce equation (10) or (11) to a single integral. As shown in Figure 4, for each angle, theta, the energy launched by the transmitting tap is distributed evenly from y_{min} to y_{max} . Since all transmitting and receiving point pairs related by the same angle theta are equivalent, the value of the integrand for this value of theta is a constant, and the integral along the receiving tap can be expressed simply as the width of the signal received. This width can be found by determining the overlap between the shadow of the transmitter for this angle, which extends from y_{min} to y_{max} , with the limits of the receiving tap, a and b . Since this received width is dependent solely on theta for a fixed transducer geometry, and theta depends on y_T , the double integral in equation (10) or (11) is reduced to a single integral over y_T as shown

$$\int_{y_T=-b}^{L-a} \int_{y_R=a}^b F(r) dy_R dy_T = \left\{ \int_{y_T=-b}^{-a} F(r) (b+y_T) dy_T + \int_{y_T=-a}^{L-b} F(r) (b-a) dy_T + \int_{y_T=L-b}^{L-a} F(r) (L-y_T-a) dy_T \right\} \quad (12)$$

where $F(r)$ represents either $u(r,t)$ or $U'(r,\omega)$, depending on the domain in which the integral is performed.

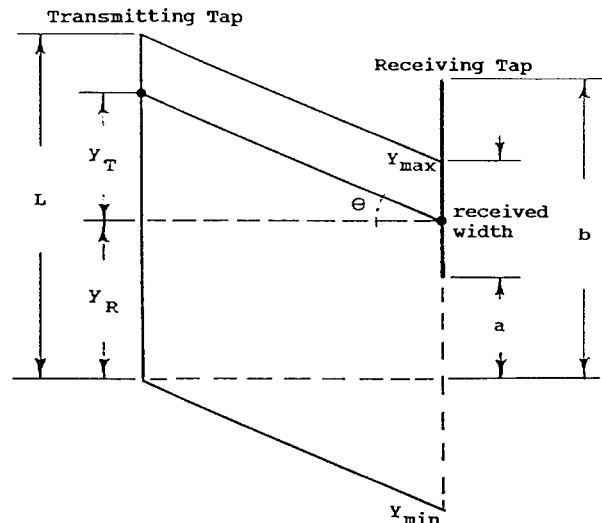


FIGURE 4

Representation of Integral Limits Used in Double Integral Reduction

Extension to Anisotropic Case

The wave equation in cylindrical coordinates used in this model assumes no variation in response with propagation direction on the substrate. Since it adds considerable mathematical complexity to include anisotropy in the wave equation itself, a first approximation to the anisotropic displacement is made by substituting surface wave velocity as a function of angle directly into the solution to the wave equation for the isotropic case. This approximation assumes that there is a small fractional deviation in velocity with a change in angle.

A look up table for $c(\theta)$ on YZ LiNbO₃ was generated using a fifteen term trigonometric expansion, as described by Anhorn, Engan, and Ronnekleiv [1]. All data presented in this paper for YZ LiNbO₃ assumes propagation along the Z axis and no power flow.

RESULTS AND DISCUSSION

Equation (9) was used to generate beam profiles at a given frequency and at several values of the scaled distance from the tap, W . W is a dimensionless parameter given by

$$W = \frac{\lambda D}{a^2} \quad (13)$$

where D is the normal distance from tap to point (as shown in Figure 1), λ is the wavelength at the frequency of interest, and the transmitting tap is of length $L=2a$.

For the isotropic case, the results obtained using the impulse driver approach compare well with those obtained using the Geometric Theory of Diffraction [6], as shown in Figure 5. For values of W less than 4, near field profiles in which most of the energy launched by the transmitting tap remains within a transverse distance roughly equal to the tap width are observed. As W increases, the beam profiles become those more characteristic of the Fraunhofer region, with a significant amount of energy spreading out transversely beyond the edges of the direct beam path.

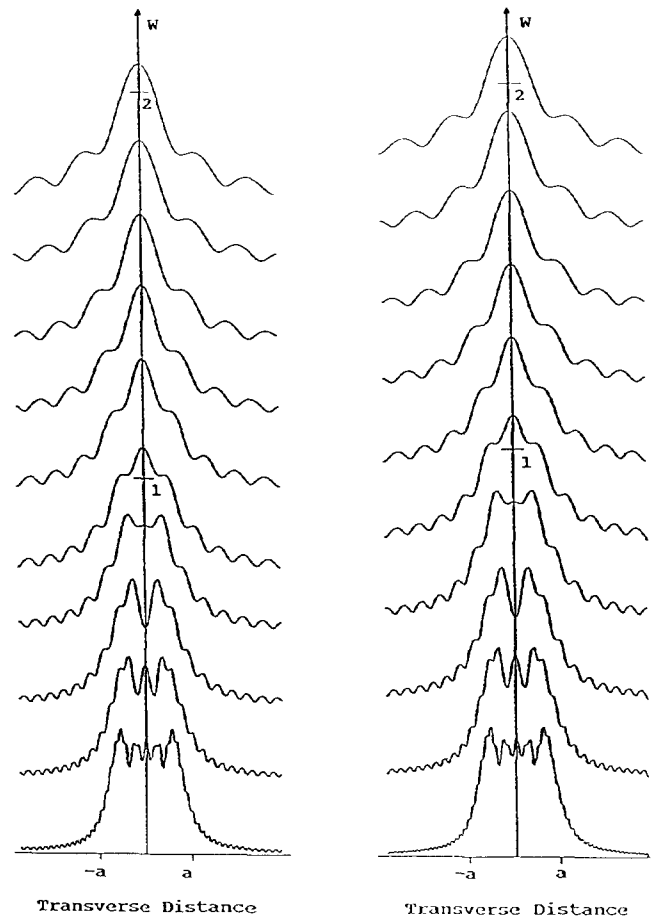
Since anisotropy does not have as great an effect on the response felt at a given point as it does on the response felt by an entire region (or tap), and since the effects of anisotropy are greatly reduced when power flow is ignored, the beam profiles generated for YZ LiNbO₃ with no power flow were very similar to those generated for the isotropic case. The profiles differ from the isotropic profiles only in the near field, where they compare favorably with those generated by Farnell for the case of parabolic anisotropy [3]. Inclusion of power flow in the analysis should have a major effect on these profiles, and is currently being developed.

Tap to tap impulse response curves were generated in the time domain from equation (10), using the double integral reduction technique. Frequency domain responses were obtained from equation (11) and inverse Fourier transformed, yielding time domain tap to tap impulse responses in good agreement with those calculated in the time domain.

Figure 6 shows the impulse response of equal

FIGURE 5

Intensity profiles at successive values of the scaled distance W for diffraction on an isotropic substrate.



(a) Profiles obtained using equation (9) [magnified by 10^6].

(b) Normalized profiles obtained using the Geometric Theory of Diffraction

length taps with no transverse offset on an isotropic substrate. As expected, no response is observed until the signal launched by the transmitting tap has had time to propagate to the receiving tap. After the impulse initially reaches the receiving tap, the response dies off rapidly. The impulse response for this configuration on YZ LiNbO₃ (ignoring power flow) is almost indistinguishable from the isotropic response.

If the taps are of equal length, but have a 50% transverse offset, the impulse responses for the isotropic case and for YZ LiNbO₃ are as shown in Figure 7. As can be seen, there is now a short time range during which direct contributions from different parts of the transmitting tap reach the receiving tap, followed by a rapid drop similar to the case of no offset. As expected, the impulse response on the anisotropic substrate is broadened

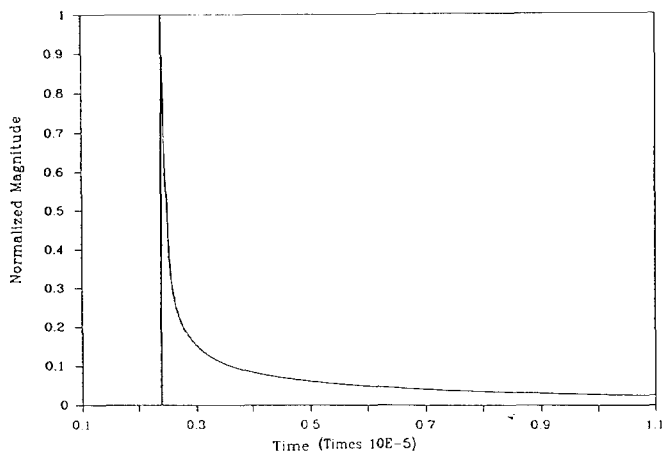


FIGURE 6

Tap to tap impulse response on an isotropic substrate. Taps are of equal length with no transverse offset.

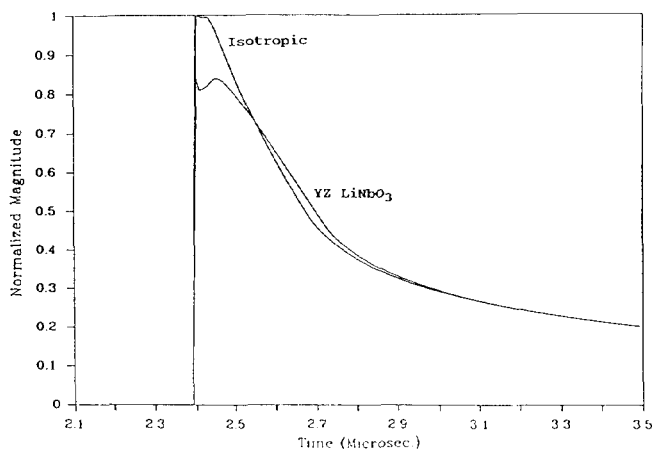


FIGURE 7

Tap to tap impulse response for equal length taps with a 50% transverse offset. Results are shown for YZ LiNbO_3 and for the isotropic case.

compared to the isotropic case. Once again, inclusion of power flow would significantly affect these results. Behavior was also investigated and impulse responses obtained for taps of arbitrary lengths and transverse positions relative to one another, yielding expected results.

CONCLUSION

This paper has presented the basic development of a new model for the analysis of SAW diffraction. The impulse driver used in the wave equation yields a response which is valid for all frequencies. Use of double integral reduction simplifies calculations considerably, while performing integrals in the time domain provides

clear insight into the geometrical basis for the responses obtained. Work is continuing to fully develop this model, including power flow and extending the analysis to complete filters. Use of this model should yield a diffraction analysis valid for broadband frequency filter responses with reduced computational time required when compared to conventional diffraction analysis.

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