

PRACTICAL IMPLEMENTATION OF COUPLING-OF-MODES THEORY FOR SAW DEVICE MODELING

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ABSTRACT

The coupling-of-modes (COM) approach to modeling surface acoustic wave (SAW) devices has been shown to be especially useful for the case of SAW transducers which have significant internal reflections when half-wavelength finger spacing is used. Implementation of the COM approach toward practical modeling of an entire SAW device has been limited thus far to the case of simple reflector arrays or transducers. Procedures are presented in this paper for integrating the COM results of Hartmann and Abbott [1] into a form useful for modeling an entire SAW device. Conversion of the 3-port P-matrix COM solutions into a standard 2-port ABCD matrix form is shown. With the resultant ABCD matrix, graphical results are shown for the full SAW device model including second order effects and matching circuitry. These results are presented for different types of SAW devices such as those with uniform, apodized, and withdrawal-weighted transducers and are compared with conventional approaches to SAW modeling. Comparison of results with actual experimental devices is also given.

Introduction

The coupling-of-modes (COM) theory has been developed in the last few years to provide a means of accurately modeling surface acoustic wave (SAW) devices in which internal transducer reflections significantly influence device response. The COM theory has the advantage of accurately modeling SAW device response with internal reflections, while providing an analytic solution for the case of simple transducers. Even for more complex devices, solutions can be obtained numerically much faster by use of the COM theory than with techniques which use lumped-element modeling of SAW transducers [2,3]. The COM theory also provides a capability of synthesizing a given SAW response with transducers which have internal reflections [4].

The COM theory was first developed for optical applications [5] and then applied to SAW device analysis by Haus [6]. It has since been used for various SAW device applications such as transducer reflection analysis [7,8] and single-phase unidirectional transducer (SPUDT) devices [9]. Recently Hartmann and Abbott [1] have combined features of the COM theory with some of the advantages of the impulse model [10] analysis of SAW transducers so that the COM theory could be applied to arbitrarily weighted transducers with internal reflections.

The results of Krimholtz [11] and Matthaei et al. [12] provide a basis for modeling an entire SAW device with reflectionless transducers. In their approach the various parts of a SAW device are separated into more basic building blocks which can be connected together to form a complete SAW device model, including the effects of the matching network. The main building block used in this model is a representation of an ideal reflectionless transducer by an ABCD matrix. By deriving analytic or numerical values for the ABCD matrix and taking into account second order effects, the entire SAW device can be modeled.

In Matthaei et al. [12] a means of including the effects of transducer internal reflections is also given. Additional building blocks are included that represent the response of the equivalent reflector arrays in the transducer. In this representation, however, the assumption is made that the transducer power is generated entirely at one point in the middle of the transducer rather than throughout the transducer.

In contrast to this, the COM theory can accurately describe the distributed nature of power transduction and reflectivity within a transducer. Therefore it can model a transducer with internal reflections without requiring separation of the reflection and transduction parts of the transducer. If an ABCD matrix expression of the results of the COM theory could be found, then the accuracy of the COM theory could be combined with advantages of the building-block approach described above.

Practical implementation of the COM theory for an entire SAW device has usually been limited to the case of simple reflector arrays or transducers. However the results of Hartmann and Abbott [1] can be used as a basis for modeling an entire SAW device with apodized transducers. In their approach they have derived hybrid S- and Y- matrices called P-matrices that can characterize a transducer response, even if the transducer is apodized. The basic approach used in this paper is to convert the 3-port P-matrix COM solutions into equivalent 2-port ABCD matrices which can then be substituted for the conventional (reflectionless) ABCD matrices of Krimholtz [10] and Matthaei et al. [12] for integration into a complete SAW device model.

Derivations

From Hartmann and Abbott [1, equation (23)] the P-matrix is defined as

$$\begin{bmatrix} b_1 \\ b_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ u_3 \end{bmatrix} \quad (1)$$

where the a - and b -terms are the S-parameter wave amplitudes, i_3 and u_3 are the current and voltage of Port 3 as shown in Figure 1.

The coefficients of the P-matrix are given as

$$P_{11} = \frac{S_H(0)}{R_i(0)} \bigg|_{S_i(L)=0} \quad P_{22} = \frac{R_H(L)}{S_i(L)} \bigg|_{R_i(0)=0}$$

$$P_{12} = P_{21} = \frac{S_H(0)}{S_i(L)} \bigg|_{R_i(0)=0}$$

$$P_{13} = \frac{P_{31}}{-2} = \frac{S_P(0)}{V_T} \quad P_{23} = \frac{P_{32}}{-2} = \frac{R_P(L)}{V_T}$$

$$P_{33} = |P_{13}|^2 + |P_{23}|^2 + jB_a(\omega) + j\omega C_0$$

in which L is the transducer length, V_T is the transducer potential, B_a is the transducer acoustic susceptance, C_0 is the transducer capacitance, R_H and R_P are the homogeneous and particular solutions respectively of the COM differential equation for the forward traveling wave, and S_H and S_P are the homogeneous and particular solutions respectively of the COM differential equation for the backward traveling wave.

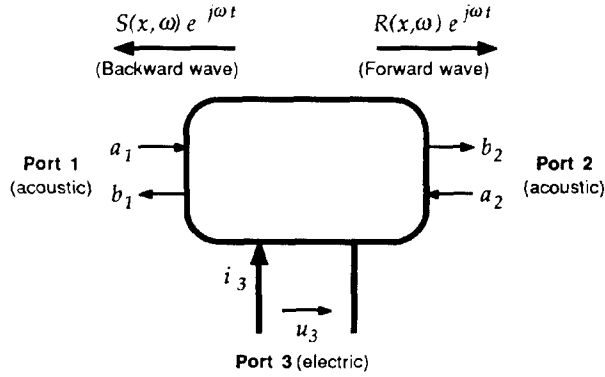


Figure 1. Representation of transducer ports

If Port 1 has a load attached to it with reflection coefficient Γ , then

$$a_1 = \Gamma b_1 \quad (2)$$

For the special case of a conventional SAW device with two transducers, Γ can usually be taken to be zero. For the general case a new 2-port P-matrix with coefficients P' can be derived corresponding to the representation shown in Figure 2. For convenience the port subscripts are changed so that $2 \rightarrow 1$ and $3 \rightarrow 2$ to give the resultant 2-port P-matrix

$$\begin{bmatrix} b_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} P'_{11} & P'_{12} \\ P'_{21} & P'_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ u_2 \end{bmatrix} \quad (3)$$

Substitution of (2) into the equation form of (1) and reduction of the equations gives the solutions for the P' coefficients in terms of the original P coefficients

$$\begin{aligned} P'_{11} &= P_{22} + \frac{\Gamma P_{12} P_{21}}{1 - \Gamma P_{11}} \\ P'_{12} &= P_{23} + \frac{\Gamma P_{13} P_{21}}{1 - \Gamma P_{11}} \\ P'_{21} &= P_{32} + \frac{\Gamma P_{12} P_{31}}{1 - \Gamma P_{11}} \\ P'_{22} &= P_{33} + \frac{\Gamma P_{13} P_{31}}{1 - \Gamma P_{11}} \end{aligned} \quad (4)$$

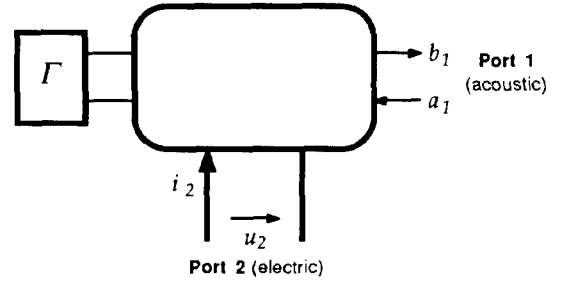


Figure 2. Two-port transducer representation

For a system with S-parameter amplitudes a and b , impedance Z_0 and current components i^+ and i^-

$$\begin{aligned} a &= \sqrt{Z_0} i^+ \\ b &= \sqrt{Z_0} i^- \end{aligned} \quad (5)$$

For any port the net current i and voltage u is defined as

$$\begin{aligned} i &= i^+ - i^- \\ u &= Z_0 (i^+ + i^-) \end{aligned} \quad (6)$$

and the ABCD matrix is defined as

$$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix} \quad (7)$$

Substitution of (5) into (3) gives in equation form

$$\begin{aligned} \sqrt{Z_0} i_1^- &= P'_{11} \sqrt{Z_0} i_1^+ + P'_{12} u_2 \\ i_2 &= P'_{21} \sqrt{Z_0} i_1^+ + P'_{22} u_2 \end{aligned}$$

Putting the Port 1 terms on the left and the Port 2 terms on the right and reducing the above equations gives

$$\begin{aligned} i_1^+ &= \frac{-P'_{22} u_2 + i_2}{P'_{21} \sqrt{Z_0}} \\ i_1^- &= \frac{-\Delta^P u_2 + P'_{11} i_2}{P'_{21} \sqrt{Z_0}} \end{aligned} \quad (8)$$

where

$$\Delta^P = P'_{11} P'_{22} - P'_{12} P'_{21}$$

Adding the two equations in (8) and using (6) gives

$$u_1 = \sqrt{Z_0} \frac{-(\Delta^P + P'_{22})u_2 + (P'_{11} + 1)i_2}{P'_{21}} \quad (9)$$

Subtracting the two equations in (8) and using (6) gives

$$i_1 = \frac{(\Delta^P - P'_{22})u_2 + (1 - P'_{11})i_2}{\sqrt{Z_0} P'_{21}} \quad (10)$$

Comparing (9) and (10) with (7) gives each term in the ABCD matrix

$$A = -\sqrt{Z_0} \frac{\Delta^P + P'_{22}}{P'_{21}}$$

$$B = -\sqrt{Z_0} \frac{P'_{11} + 1}{P'_{21}}$$

$$C = \frac{\Delta^P - P'_{22}}{\sqrt{Z_0} P'_{21}}$$

$$D = \frac{P'_{11} - 1}{\sqrt{Z_0} P'_{21}}$$

Experimental Results

Integrating the above ABCD-matrix coefficients into the complete modeling program described earlier gives a direct comparison of experimental results with theory. The experimental results will be compared with both the COM theory as implemented with the ABCD matrix solution derived above, and the approach of Matthaei et al. [12] (hereafter called the lumped model) in which the reflection and power transduction in a transducer are separated.

Four basic SAW device designs are evaluated: (A) unapodized transducers, (B) unapodized to overlap-weighted transducers, (C) unapodized to withdrawal-weighted transducers, and (D) a single unapodized transducer with many fingers and a narrow aperture. All of the devices had a nominal center frequency of 300 MHz with two fingers per wavelength in all transducers. Non-reflecting dummy fingers are added at the ends of each transducer in order to minimize transducer end effects which are not fully modeled. A Gaussian-like weighting function with no time sidelobes was used for all weighted transducers. All devices were fabricated on a substrate of ST-cut quartz with a transducer center-to-center separation of 121 mils (except for Device D).

Table I summarizes the characteristics of each transducer while Table II shows how each transducer was used in the different devices.

TABLE I: TRANSDUCER DESCRIPTION

Transducer	Length	Aperture	Weighting
a	50 λ	100 λ	uniform
b	50 λ	100 λ	overlap
c	50 λ	100 λ	withdrawal
d	300 λ	10 λ	uniform

TABLE II: DEVICE DESCRIPTION

Device	Transducer 1	Transducer 2
A	a	a
B	a	b
C	a	c
D	d	(none)

In order to evaluate the accuracy of the model, several test devices were fabricated using different metal thicknesses. The electrode periodicity was calculated from the free-space substrate velocity with a nominal center frequency of 300 MHz. In this way frequency skewing effects resulting from different metal thickness could be compared. Parallel bond wires were used for assembly of all devices to minimize series inductance. However for best accuracy a series inductance and parallel capacitance was included for all modeled results to account for stray parasitics. The estimated series inductance due to bonding was 8 nH for devices A-C and 4 nH for device D with an estimated parallel capacitance of 0.5 pF due to package capacitance. Device D also had the estimated bus bar resistance, capacitance, and inductance included for the modeled results.

Time-domain gating was used to eliminate the effects of any electromagnetic feedthrough in all devices. For most of the results shown, except where otherwise noted, all multiple-transit signals were also eliminated by time-domain gating in order to show a clearer comparison of the experimental and modeled results. Except where otherwise noted, a solid line is used in all plots to denote modeled results while the heavy dashed line is used to denote experimental results.

Figure 3 illustrates the excellent agreement for both the insertion loss and general shape of the response between COM modeling and experimental results for device A. A relatively heavy metal thickness of 2000 Å (corresponding to a thickness-to-wavelength ratio of 1.9%) was used to illustrate the accuracy of the COM theory even for large finger reflection coefficients.

Figure 4 shows the slightly poorer agreement with experiment for the lumped model. In both models the frequency skewing due to changes in both the velocity and reflection coefficients agree closely with experiment.

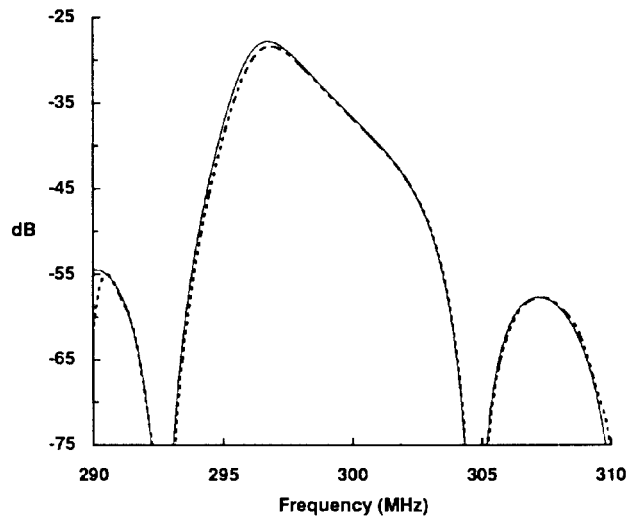


Figure 3. COM vs. experiment for device A (2000 Å)

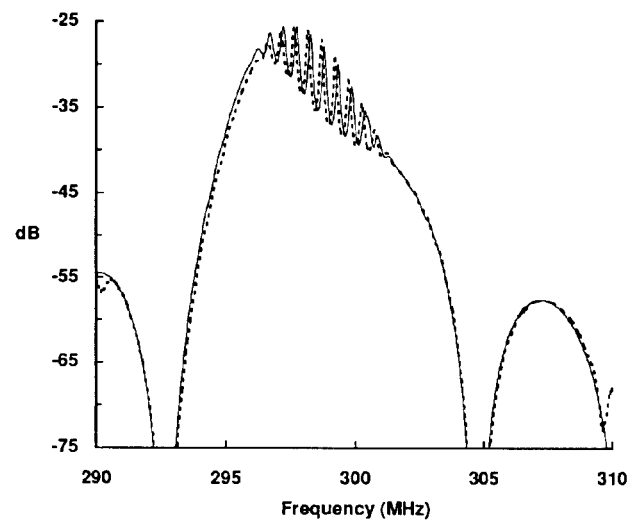


Figure 5. COM vs. experiment for device A (2000 Å)

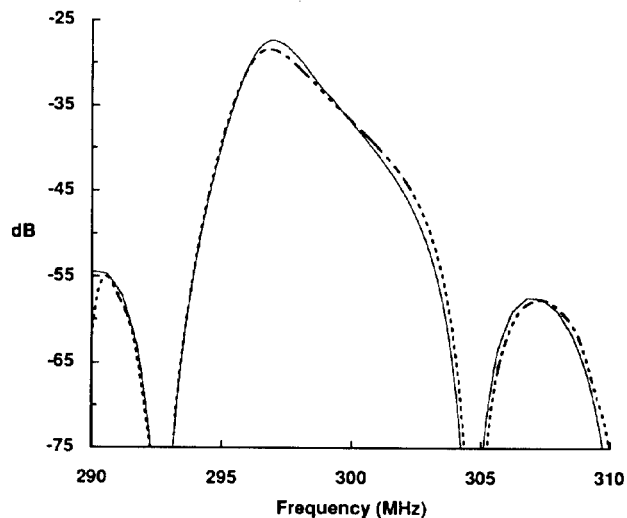


Figure 4. Lumped model vs. experiment for device A (2000 Å)

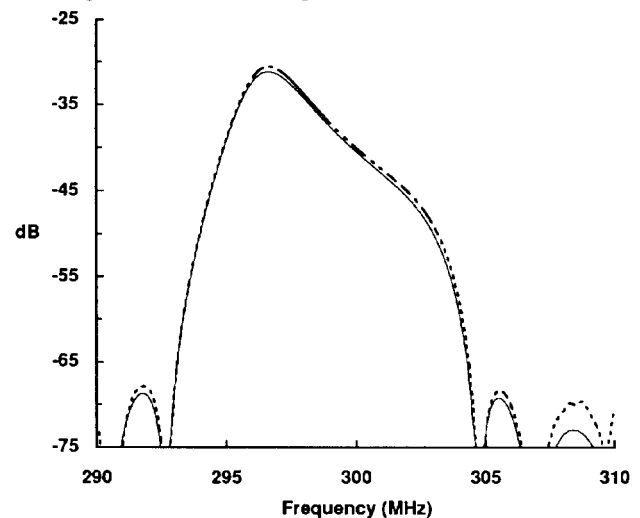


Figure 6. COM vs. experiment for device B (2000 Å)

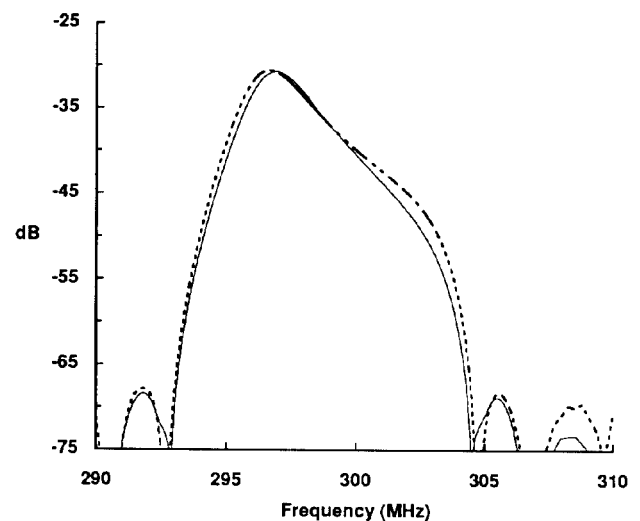


Figure 7. Lumped model vs. experiment for device B (2000 Å)

Figure 5 shows the same device as in Figure 3, except that the multiple-transit signals are included. This figure shows that the COM model can accurately handle both regenerative and mechanical reflection effects.

The response of device B which has an overlap-weighted transducer is shown in Figure 6. Agreement between theory and experiment is again quite good even without channelization or "mode expansion" [1], which should be implemented to obtain the most accurate results. Figure 7 shows the same device in comparison with the lumped model. The lumped model was also evaluated with the overlap-weighted transducer divided into ten channels, but the results were almost identical to those in Figure 6. Again it can be seen that there is better agreement with experiment for the COM model than with the lumped model even without channelization.

Figure 8 shows the excellent agreement between the experimental and COM response for device C which has a withdrawal-weighted transducer. The effects of different transducer metal thickness (1000 Å, 1500 Å, and 2000 Å) can be seen in the COM-modeled responses shown in Figure 9. It can be clearly seen in this figure that both the amplitude and frequency skewing tend toward lower frequencies as the metal thickness is increased.

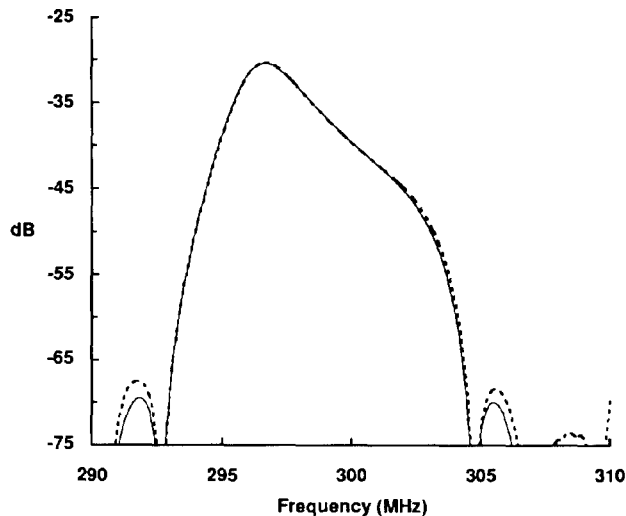


Figure 8. COM vs. experiment for device C (2000 Å)

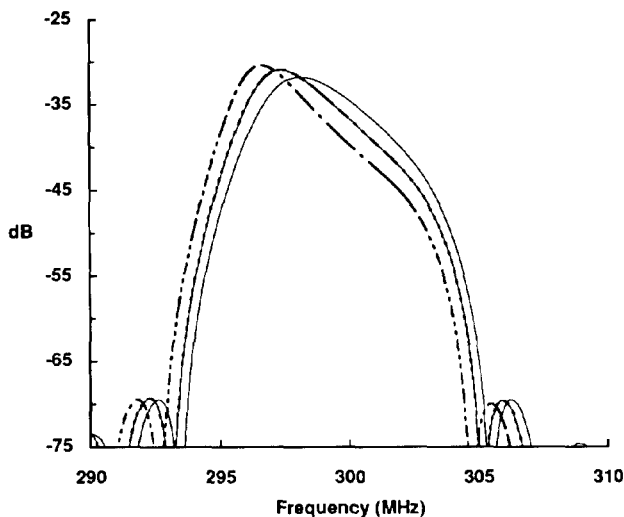


Figure 9. COM model for device C with different metal thicknesses (1000 Å, 1500 Å, 2000 Å)

Device D had only a single transducer with the result that only the conductance is shown for this device. The transducer in this device also had a relatively large number of fingers (300) which would be expected to cause a large distortion in the response due to internal reflections. A comparison of the different modeled conductance responses with different metal thicknesses (1000 Å, 1500 Å, and 2000 Å) is shown in Figure 10. It shows that the skewing to lower frequencies is more pronounced for thicker metal. For reference the

lumped-model response of the same device with 1000Å metal thickness, but with no internal reflections, is shown by the heavier solid line in the same figure.

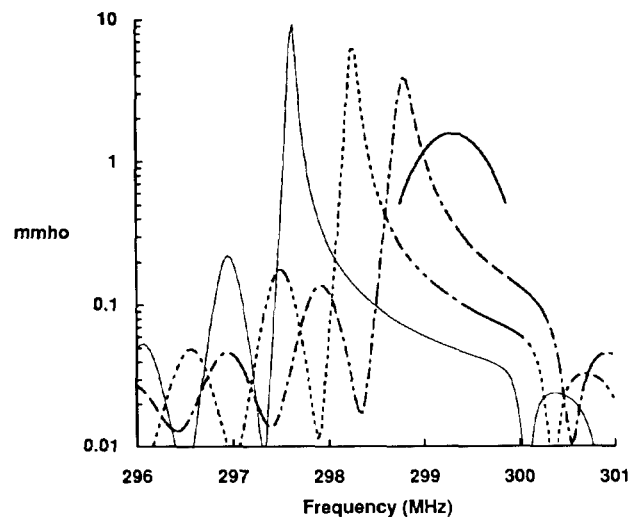


Figure 10. Conductance of COM model for device D with different metal thicknesses (1000 Å, 1500 Å, 2000 Å)

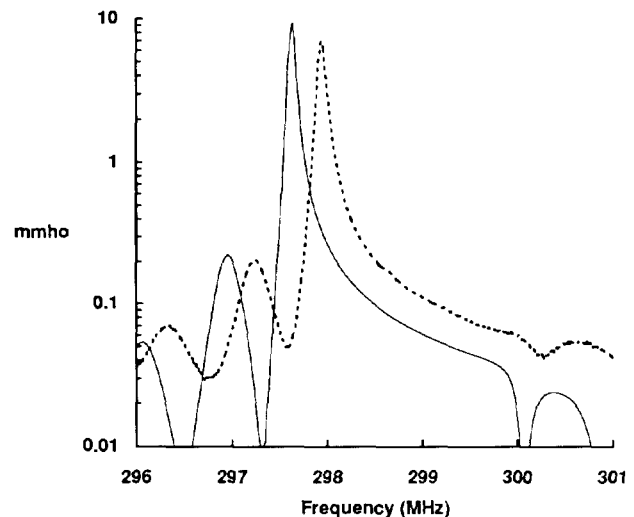


Figure 11. Conductance of COM vs. experiment for device D (2000 Å)

The experimental conductance of device D is compared in Figure 11 with that predicted by the COM model for a metal thickness of 2000 Å. There is good agreement with experiment, except some deviation in the frequency offset of the response. For the case of the large amount of frequency offset shown in the figure, the exact amount of offset is very sensitive to the exact velocity and reflection coefficients used in the model. Figure 12 compares the same experimental data with the conductance predicted by the lumped model response. It can be clearly seen in the figure that the shape of the lumped-model response deviates much more from experiment than the COM-modeled response of Figure 11.

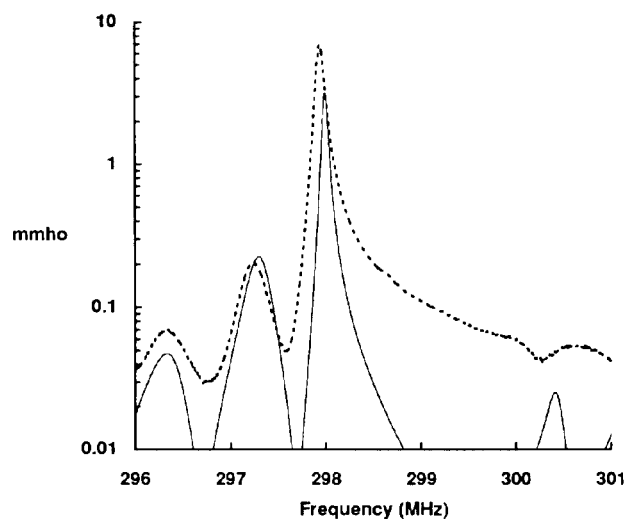


Figure 12. Conductance of lumped model vs. experiment for device D (2000 Å)

Experimental data was also taken for devices with metal thicknesses of 1000 Å and 1500 Å, but are not presented in this paper. The data for 2000 Å thickness would generally represent the "worst case" and would more clearly show skewing of the responses due to internal transducer reflections. The agreement with experiment for the COM model is generally even better for thinner metal thicknesses.

Conclusions

Application of the COM theory toward modeling of an entire SAW device has been demonstrated. Derivation of the coefficients of the ABCD transmission matrix from COM theory has been given which can then be utilized in a more complete SAW device model. Experimental results for various device configurations and metal thicknesses have been shown for comparison with predicted model results. In general there is close agreement between experiment and the implementation of COM theory reviewed in this paper.

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