RESEARCH REPORT | 26 SEPTEMBER 2026

What Is a Centimetre of Water Worth?

The nonlinear economics of navigable depth on the IJssel

Research report | Transport economics and scenario analysis

A centimetre of navigable depth has no universal economic price. Its value depends on loading, capacity, alternatives and whether route availability changes.

REPORT DATE	26 September 2026
EVIDENCE CUT-OFF	26 September 2026 report release
PRICE BASIS	2024 euros
REFERENCE ARTICLE	What Is a Centimetre of Water Worth?

2.50 to 2.51 metres	1.45 to 1.46 metres	1.44 to 1.45 metres
About €250	**€3,000**	**€21,760 Only if access is restored**

Illustrative savings per 10,000-tonne planning period, in 2024 euros; not measured IJssel-wide losses.

Executive summary

A centimetre of navigable depth has no universal economic price. Its value depends on where the centimetre is added, which vessels and cargoes are affected, how long it is available, and whether it changes the transport options that are feasible. The relevant quantity is the reduction in the cost of delivering a specified amount of freight after shippers have adjusted loading, departures and transport alternatives.

The IJssel provides a concrete application. Rijkswaterstaat's September 2026 notices identify a low-water operating threshold and a subsequent directional restriction. The economic analysis must nevertheless distinguish three things: the physical effect of depth on payload, the operational decision to allow a movement, and the welfare consequences of changing either one. A published closure trigger is not proof that an additional centimetre will restore access.

This report develops a constrained cost-minimisation model and a reproducible numerical example. **The numerical results are conditional scenarios, not measured IJssel-wide losses.** For an illustrative 10,000-tonne planning period, a depth increase from 2.50 to 2.51 metres reduces allocated transport cost by about €250. From 1.45 to 1.46 metres, the saving is €3,000 because a departure-capacity constraint binds. A change from 1.44 to 1.45 metres generates €21,760 only under the additional assumption that it restores route availability. All amounts are in 2024 euros.

The principal finding is not simply that water becomes more valuable as it becomes scarce. **Adaptation can cap its value, capacity constraints can change the shape of the curve, and an operating-regime change can create a discrete benefit.** Under a cheaper transport alternative, the illustrative reopening benefit disappears altogether. Consequently, the transport-cost curve is a useful input to infrastructure appraisal, but it is neither a complete social valuation of water nor a spending recommendation.

EVIDENCE AND OPERATING CONTEXT

1. The event and the existing evidence

1.1 A defined restriction, not the closure of the entire river

Rijkswaterstaat announced that one-way operation would begin on 25 September 2026 between the Twentekanaal mouth near Zutphen and the IJsselkop near Arnhem. Upstream movements toward Arnhem remain permitted; the opposite direction is prohibited. The notice attributes the restriction to a narrowing navigable channel and describes a diversion

via the Waal, Amsterdam-Rijnkanaal and northern lakes of up to 300 kilometres. That description does not establish the additional distance for every origin-destination pair.[1]

The earlier warning specified a minimum measured depth, or *Minst Gepeilde Diepte* (MGD), below 1.45 metres as the trigger. It did not supply a separately verified reopening rule. The numerical cutoff used below therefore represents a counterfactual operating-state assumption, not navigational advice.[2]

1.2 What the historical evidence establishes

The regional study of the 2018 drought reports an IJsselkop-Twentekanalen target depth of 2.50 metres and a minimum MGD of 1.45 metres. It records five months of reduced loading, with loads at the extreme falling to one-third of vessel capacity, additional movements, and some substitution to road transport subject to capacity limits. These are historical observations, not a loading curve for every vessel or a current traffic count.[3]

The underlying economic question is established in the literature. Using Rhine voyage data from 2003-2005, Jonkeren, Rietveld and van Ommeren found that water levels affected load factors and freight prices per tonne, while the estimated effect on price per trip was close to zero. Their study is an empirical precedent for the mechanism, not an estimate that can be transferred directly to the IJssel.[4]

Later research explicitly models changes in the competitive position of inland shipping relative to other modes. That supports treating transport choice as an adjustment margin rather than assuming that all freight stays on the same route regardless of cost.[5]

The contribution here is the application of those principles to a directional operating threshold, with the within-regime value of depth separated from the value of changing regimes.

MEASUREMENT AND ECONOMIC BOUNDARY

2. Define the centimetre before pricing it

2.1 Depth, draft and water level are different variables

A gauge reading is an elevation relative to a datum. Navigable depth is a distance between the water surface and the relevant bed or obstruction. Vessel draft is the portion of the vessel below the waterline. They cannot be substituted for one another without a physical mapping.

For the analysis, let h be the controlling route depth in metres. A simplified loading relationship is:

$$q(h) = min\{q_{max}, \alpha\, max(h - d_0 - u, 0)\}.$$

Here $q(h)$ is cargo tonnes per movement; q_{max} is maximum payload; d_0 is effective non-cargo draft; u is an under-keel allowance; and α is the incremental payload per metre of draft. This is a modelling approximation. A calibrated application would use vessel measurement certificates and operating allowances that account for the relevant hull, consumables, squat and safety conditions.

The equation makes the mechanism explicit: additional depth has a payload benefit only when draft is the binding limitation. There is no further payload benefit once the vessel is at its cargo, volume or certified loading limit. Nor does deeper water at a non-controlling location necessarily change the allowable payload over the complete route.

2.2 State the economic and time boundaries

Let $C(h, s; Q)$ be the minimum transport cost of delivering freight quantity Q under operating state s. The function is evaluated after transport choices have been re-optimised. State s includes route availability and any other operational constraints held fixed in a scenario.

Two different valuations follow:

$$V_{1cm}(h, s; Q) = C(h, s; Q) - C(h + 0.01, s; Q),$$

and

$$\Delta C_{regime}(h; Q) = C(h, restricted; Q) - C(h, available; Q).$$

The first is a finite one-centimetre benefit within a stated regime. The second is a same-depth comparison between operating regimes. Only an intervention that actually changes the regime can claim the second benefit.

This report normalises the numerical exercise to **10,000 tonnes per planning period**. It does not assume that the period is one day, and the departure limit applies to that same period. A river-wide daily estimate requires observed or modelled daily freight requirements, operating capacity and route choices. Annual traffic divided by 365 would not establish the affected low-water flow.

THE ECONOMIC MODEL

3. The economic model

3.1 Constrained cost minimisation

The smallest useful model has a shallow-water route and an alternative transport chain. The latter could represent another water route, road, rail, or a combination, provided it is genuinely feasible for the cargo concerned.

Let n denote shallow-route trip equivalents, y tonnes carried by the alternative, K cost per shallow-route movement, g alternative cost per tonne, and N available departure equivalents. Let z equal one when the route is available and zero when it is unavailable. The model is:

$$C(h, z; Q) = \min_{n, y} \{K n + g y\},$$

subject to

$$q(h) n + y \geq Q, 0 \leq n \leq N z, y \geq 0.$$

This is a partial-equilibrium cost model: freight demand is fixed, and prices or resource costs outside the specified transport choices are not determined within it. It is deliberately not a full model of the Dutch economy.

If the shallow route costs less per tonne than the alternative and is available, it carries the smaller of total demand and its available payload capacity. Otherwise the alternative carries the freight. The solution therefore permits both substitution and an explicit capacity constraint; it does not force the cost curve to diverge at low depth.

The constant, unlimited fallback is a simplifying assumption, not an assertion about road or rail capacity. If alternatives also become congested, their capacity limits and rising marginal costs belong in the model. If freight cannot be moved immediately, waiting, inventory and unmet-demand costs require additional choices and constraints.

3.2 Why a nonlinear curve emerges

When all freight uses the shallow route and departure capacity is slack, cost is:

$$C(h)=\frac{K\,Q}{q(h)}.$$

For the linear, uncapped portion of the loading relationship, writing $a=d_0+u$ gives:

$$C(h)=\frac{K\,Q}{\alpha(h-a)},\quad -\frac{dC}{dh}=\frac{K\,Q\,\alpha}{q(h)^2}.$$

The inverse-payload term creates the nonlinearity. Losing ten tonnes of capacity from a lightly loaded vessel has a larger cost effect per tonne delivered than losing the same ten tonnes from a heavily loaded vessel. This is a result of the specified cost structure, not an assumption that every centimetre must have a larger value than the previous one.

Once all available departures are used and some cargo moves by the alternative:

$$C(h)=N\,K+g\left[Q-N\,q(h)\right).$$

Within that interval, the benefit of an extra centimetre is $0.01\,g\,N\,\alpha$. Departure expenditure is unchanged; the extra payload displaces alternative transport. When the alternative becomes cheaper for all freight, or the shallow route remains unavailable, its immediate depth benefit can instead be zero.

3.3 A shadow price, with a specific meaning

The Lagrange multiplier on a binding constraint measures the value of relaxing that constraint inside the model. It is a *shadow price*: it need not be a price paid in a market. Appendix A derives the relationship between the payload constraint and the depth derivative using the envelope theorem. This mathematical treatment applies at differentiable points; a discrete access change must be valued by comparing complete solutions, not by extending a local derivative through the jump.[6]

Because the numerical costs below are allocated operating-cost proxies, their shadow values inherit that accounting boundary. They are not automatically social shadow prices. That distinction matters before drawing conclusions about public spending.

SCENARIO ANALYSIS

4. A transparent numerical exercise

4.1 Inputs and provenance

KiM's 2026 publication provides freight-cost data through 2024. The inland-shipping table gives a total operating-cost benchmark of €28.31 per kilometre for an M8 vessel carrying agricultural and food products, and identifies the M8 class at 2,700 tonnes. These are class and accounting benchmarks, not a quoted IJssel shipment.[7,8]

Table 1. Inputs to the numerical scenario

Parameter	Value	Evidence status
Freight requirement	10,000 t / period	Normalisation; not observed traffic
Maximum payload	2,700 t	Published M8 class benchmark
Payload slope	1,000 t / m	Assumed; equivalent to 10 t / cm
Effective non-cargo draft	0.80 m	Assumed; sensitivity tested
Under-keel allowance	0.20 m	Assumed; not safety guidance
Allocated movement cost	€5,662	€28.31/km benchmark x assumed 200 km
Available departures	20 / period	Assumed aggregate limit
Alternative transport cost	€15 / t	Assumed constant, unlimited fallback
Route-state switch	1.45 m	Sourced closure trigger; assumed availability switch

All monetary inputs and outputs are in 2024 euros. The 200-kilometre movement length is an illustration, not a measured route, round trip or diversion. The operating-cost coefficient includes allocated fixed and personnel costs; these are not added again as separate time costs.

The exercise intentionally leaves the crucial assumptions visible. It combines a published cost benchmark with assumed loading and substitution parameters; that is different from calibrating those parameters to observed vessels and shipments. The alternative cost is defined at the same transport boundary. Cargo inventory costs and wider economic effects are outside both alternatives in this example.

4.2 The resulting curve

Table 2. Optimised costs and one-centimetre benefits

Starting depth	Route state	Payload if available	Total cost / period	Saving from +1 cm
3.70 m	Available	2,700 t	€20,970	€0
2.50 m	Available	1,500 t	€37,747	€250
2.00 m	Available	1,000 t	€56,620	€561
1.70 m	Available	700 t	€80,886	€1,139

Starting depth	Route state	Payload if available	Total cost / period	Saving from +1 cm
1.50 m	Available	500 t	€113,240	€2,220
1.45 m	Available	450 t	€128,240	€3,000
1.44 m	Unavailable	440 t	€150,000	€21,760*
1.40 m	Unavailable	400 t	€150,000	€0

Source: model calculations using Table 1. Rounded to the nearest euro. Each row is a separate scenario with the same freight requirement. An asterisk marks a change in the assumed operating state, not an ordinary within-regime marginal benefit.

At 2.50 metres, the model needs 6.67 trip equivalents. One more centimetre raises payload from 1,500 to 1,510 tonnes and reduces the required movement frequency. At 1.45 metres, all 20 departures are used: they move 9,000 tonnes, with 1,000 tonnes using the alternative. One extra centimetre adds 10 tonnes on each of 20 departures, displacing 200 tonnes of alternative transport. At €15 per tonne, that is €3,000.

Below the assumed access threshold, none of the modelled directional freight uses the shallow route. Raising depth from 1.40 to 1.41 metres changes neither the operating state nor fallback cost, so its immediate benefit is zero. This does not imply that water has no ecological value, no benefit for permitted traffic, or no value in reducing future disruption risk. It is a statement about this particular cost boundary and state.

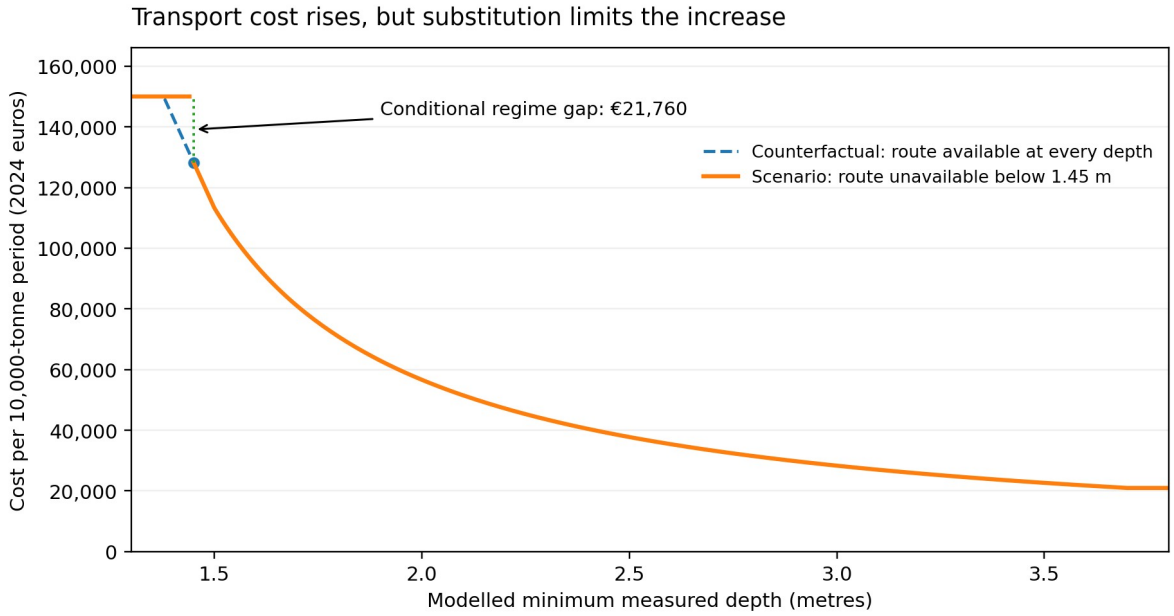


Figure 1. Optimised transport cost. The restriction changes the available choice set; the dashed curve holds route availability open as a counterfactual. Both curves allow substitution and a departure limit. Source: model calculations; Table 1 assumptions.

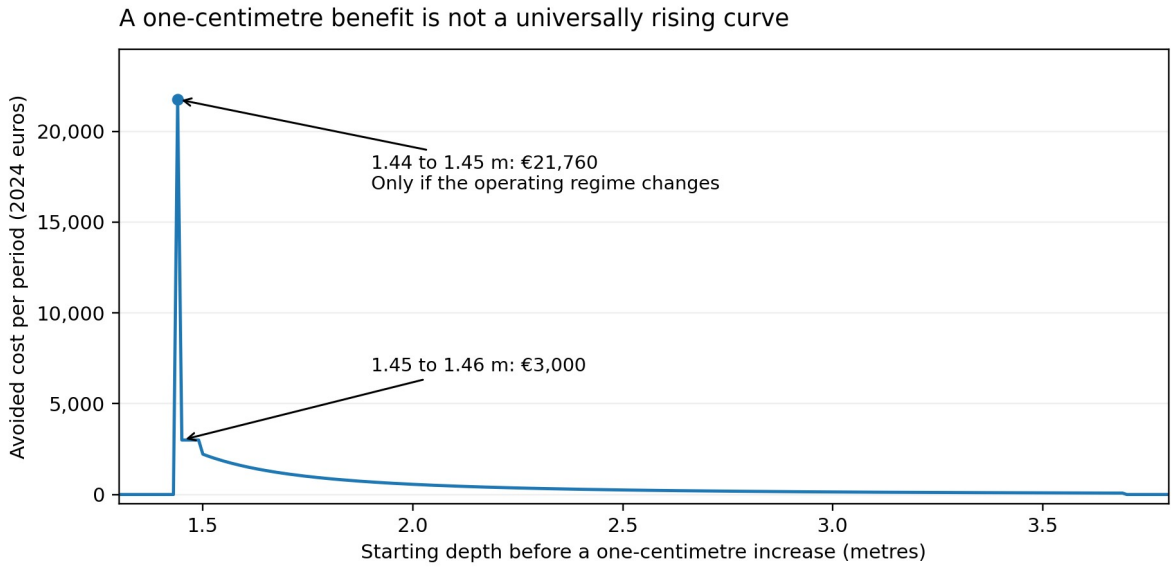


Figure 2. Finite one-centimetre savings on a one-centimetre grid. The peak is conditional on a regime change. Connecting lines guide the eye; they are not an estimated smooth derivative. Source: model calculations; Table 1 assumptions.

4.3 Whole trips and short-run cash expenditure

The continuous model treats movements as aggregate frequencies over a planning period. A particular consignment requires whole trips. For the same 10,000 tonnes, seven movements cost €39,634 at both 2.50 and 2.51 metres in an integer version of the model. The one-centimetre cash saving for that isolated consignment is zero, even though the continuous cost allocation falls by about €250.

This is an economically meaningful indivisibility, not numerical noise. Depth can improve utilisation before it eliminates a departure. Moreover, allocated crew and capital costs may not be

avoidable during a brief disruption. A measured short-run cash-cost analysis must identify which expenditures and opportunity costs actually change, rather than treating the entire accounting benchmark as marginal expenditure.

SENSITIVITY AND ROUTE CHOICE

5. What determines the size of the threshold effect?

5.1 Reopening has value only when the restored option is useful

At a fixed depth h_c, the same-depth benefit of making the route available in this model is:

$$\Delta C_{regime} = min\{Q, N\,q(h_c)\}\,max\left\{g - \frac{K}{q(h_c)}, 0\right\}.$$

It is the amount of freight that can benefit multiplied by the cost advantage per tonne. This prevents the analysis from crediting a route with a large benefit merely because it has become legally available.

Table 3. Same-depth regime benefit at 1.45 metres

Available departures / period	Alternative €10/t	Alternative €15/t	Alternative €20/t	Alternative €25/t
10	€0	€10,880	€33,380	€55,880
20	€0	€21,760	€66,760	€111,760
40	€0	€24,178	€74,178	€124,178

Source: model calculations. These are alternative parameter scenarios, not probability bounds or confidence intervals. The calculation compares route availability at the same depth and does not assume an engineering intervention can deliver it.

At €10 per tonne, the alternative is already cheaper than the restored shallow route, so reopening has no transport-cost benefit within this specification. More departure capacity also ceases to add value once all eligible freight can use the restored route. The benefit cannot be scaled indefinitely by multiplying it by a larger vessel count.

The loading relationship is equally influential. Holding the other inputs fixed, an effective non-cargo draft of 0.70 metres yields an illustrative regime benefit of about €47,055; at 0.80 metres it is €21,760; at 0.90 metres it is zero because the shallow-route cost exceeds the €15 fallback. The magnitude is therefore not robustly identified by the operating threshold and cost benchmark alone.

5.2 A diversion changes more than distance

The correct comparison holds delivered freight constant, not vessel count. If direct and diversion routes permit different payloads, a simplified unconstrained comparison is:

$$\Delta C = Q\left(\frac{k\,L_1}{q_1} - \frac{k\,L_0}{q_0}\right).$$

Subscripts zero and one denote direct and alternative routes. Different lock delays, speeds, terminal requirements, empty positioning and fleet-cycle times would need explicit treatment beyond this distance-only expression. A longer route can permit a different payload and require a different number of movements. The route comparison therefore cannot be replaced by “300 kilometres times cost per kilometre times all vessel passages”.

A spatial model would also track the controlling depth along every feasible path. If another section remains equally shallow, deepening only one bottleneck may add no usable draft. Improving both can have value even when improving either alone does not. This complementarity is why a centimetre at a particular location is not interchangeable with a centimetre everywhere in the river.

WELFARE ACCOUNTING

6. Transport costs are not the social value of water

6.1 Resource losses, payments and welfare

Three accounts must remain separate. A shipper’s invoice records a private payment. An operator’s cost account records expenditure and allocated costs. A social welfare account records changes in real resource use and in the benefits people receive, with a stated geographical boundary. Dutch cost-benefit guidance takes that wider societal perspective and explicitly includes effects, risks and uncertainties rather than only financial cash flows.[9]

For example, a freight surcharge paid to an operator inside the chosen welfare boundary is not automatically an additional social loss equal to the payment. Some of it may compensate for extra fuel and vessel time; some may be a transfer of surplus or a scarcity rent. Conversely, displaced use of a fully occupied vessel has an opportunity cost even when no extra wage payment is made. Cross-border incidence matters when the welfare boundary is national rather than international.

A welfare extension would replace accounting proxies with relevant resource and opportunity costs, then consider cargo delay, reliability, emissions, congestion, production effects and other water uses. It must avoid counting the same effect twice. Freight appraisal guidance specifically warns about overlap between vehicle costs and the valuation of transport time.[10]

The value of delayed cargo is not the value of destroyed cargo. Likewise, reduced output should not be valued automatically at gross sales: avoided production costs, later catch-up and

substitution can change the net loss. These are separate empirical questions, not multipliers to attach indiscriminately to transport expenditure.

6.2 Fixed freight demand is a benchmark, not a universal assumption

The numerical model treats all 10,000 tonnes as needing delivery within the period. If delivery can be postponed, stocks drawn down, or sourcing changed, fixed-demand transport expenditure is no longer a complete welfare measure. One extension is to choose delivered quantity D as well as transport:

$$W(h) = \max_{D} \{ U(D) - C_{resource}(h; D) \},$$

where $U(D)$ captures the net economic benefit of deliveries at the specified time. This introduces the value of unmet or delayed demand rather than pretending that unlimited alternative capacity is always available. Without a defensible demand or shortage-cost relationship, no numerical welfare estimate follows from this extension.

6.3 A centimetre of depth is not a cubic metre of water

The calculation values a change in transport conditions. It does not yet value a unit of river discharge, stored water or dredged sediment. Raising water level and lowering a local obstruction may have different effects on usable width, flow, downstream conditions and other users.

A conversion to euros per cubic metre requires a hydraulic response function: how a release or allocation changes controlling depth over time, together with the opportunity cost of water elsewhere. Multiplying an assumed surface area by one centimetre would describe a static volume change, not the sustained water requirement of a flowing river.

For an intervention A, the shipping component of a discounted benefit calculation is the expected difference between complete transport-cost paths with and without the intervention. Social appraisal would then add or subtract effects on other uses and the intervention's resource costs. This report supplies a transport-model structure for that exercise; it does not identify a total social benefit or a preferred intervention.

CALIBRATION AND LIMITS

7. Duration, uncertainty and empirical identification

7.1 Benefits depend on the depth path

A centimetre available for one afternoon is not equivalent to a centimetre maintained through a prolonged disruption. Freight demand, vessels, inventories and alternative capacity can change over the episode. Benefits should therefore be evaluated period by period, with

capacity carried consistently through vessel cycles, rather than multiplying the largest observed or modelled one-centimetre value by every day of a drought.

Uncertainty also changes threshold valuation. To isolate the mechanism, suppose the cost function near a threshold contains a locally constant penalty J whenever uncertain depth H falls below h_c. If an intervention raises depth by δ, its expected avoided threshold penalty is:

$$J\,Pr\left(h_c - \delta \le H < h_c\right).$$

This formula is a local illustration, not an estimated probability model for the IJssel. It shows why a large conditional disruption cost need not imply an equally large expected benefit from a small intervention. The intervention must move the system across the relevant threshold in states that actually occur. A separate reopening threshold, measurement uncertainty or operational discretion would require a richer state model.

7.2 What a measured IJssel curve requires

Table 4. Data needed to estimate, rather than illustrate, the curve

Data block	Required observation	Economic use
Hydraulic conditions	Reach-level MGD, timing, channel geometry and uncertainty	Map depth changes to usable routes and payload
Operating state	Start/end times, direction, vessel restrictions and reopening conditions	Separate physical scarcity from access changes
Freight and vessel movements	Origin, destination, cargo, actual load, vessel type and alternatives	Identify the affected denominator and selection
Vessel loading and cycles	Measurement certificates, allowances, loaded/empty legs and turnaround	Estimate payload functions and available departures
Comparable transport costs	Route distances, time, variable costs, capacity and alternative quotes	Re-optimise within a consistent cost boundary
Demand and disruption	Delivery deadlines, stocks, postponed orders and production response	Distinguish higher expenditure from welfare loss

AIS tracks alone do not establish cargo tonnage, and passage counts do not establish how much freight would have used the prohibited direction without the restriction. Both need supporting load and origin-destination information.

A structural estimate would first reproduce observed route choices, payloads and costs, then solve the same freight problem under small depth perturbations. Back-testing against separate low-water episodes would test whether the inferred curve predicts adjustments outside its calibration sample. Survey errors and parameter uncertainty should be propagated through the model, not disguised by reporting many decimal places.

A causal estimate of the restriction is a different task. A before-and-after comparison can confound the operating rule with continuing drought, fuel costs, commodity demand and network-wide disruption. A threshold-based research design would need evidence that treatment changes reliably at the cutoff and that other determinants evolve smoothly nearby. Anticipation, separate reopening conditions and spillovers to comparison routes could invalidate a simple regression-discontinuity interpretation. Observing a published threshold alone does not solve identification.

There is also selection: vessels that stop using the route disappear from its observed freight-price sample. Estimating only the cost of surviving movements can understate the burden on diverted or unserved cargo. The affected freight set must therefore be defined before observing its post-restriction transport choice.

8. Conclusion

The useful answer to “What is a centimetre of water worth?” is a state-contingent cost curve, not a universal price. Within a route and operating regime, depth changes payload and the allocation of scarce departures. Across regimes, it changes the feasible transport network. Substitution, capacity and whole-trip decisions determine whether those changes produce rising marginal benefits, plateaus, discrete jumps or no immediate saving.

The numerical exercise demonstrates these mechanisms and makes their assumptions reproducible. It does not establish the observed daily economic loss on the IJssel. Its conditional €21,760 regime benefit can disappear under a cheaper alternative or a different loading curve. That sensitivity is a substantive result: the local threshold alone does not identify the economic magnitude.

TECHNICAL DERIVATION

Appendix A. Shadow prices and the envelope calculation

For the two-option model, write the Lagrangian as:

$$L = K\,n + g\,y + \lambda\left[Q - q(h)\,n - y\right] + \mu\left[n - N\,z\right],$$

with non-negative multipliers λ and μ. The first multiplier is the marginal model cost of an additional tonne required for delivery; the second is the value of an additional departure equivalent when the departure limit binds. The Kuhn-Tucker conditions include:

$$K - \lambda\,q(h) + \mu \geq 0,\; g - \lambda \geq 0,$$

with equality for an actively used choice, and the complementary-slackness conditions:

$$\lambda\left[Q - q(h)\,n - y\right] = 0,\; \mu\left[n - N\,z\right] = 0.$$

For a fixed operating state, fixed K and fixed departure limit, the local envelope result is:

$$-\frac{\partial C}{\partial h} = \lambda\,n\,q'(h).$$

This is in euros per metre per planning period. Multiplying by 0.01 gives a local euros-per-centimetre approximation. The exact one-centimetre benefit is the finite difference of the re-optimised cost function, especially near kinks.

If only the shallow route is used and departure capacity is slack, $\mu=0$ and $\lambda=K/q$. Substitution gives the derivative KQq'/q^2. If both routes are used and the departure limit binds, $\lambda=g$ and $\mu=gq-K$. The derivative is then gNq'. These are the economic mechanisms behind the two within-regime sections of the numerical curve.

If movement cost and available departures also depend smoothly on depth, the available-regime expression becomes:

$$-\frac{\partial C}{\partial h}=\lambda n q'(h)+\mu N'(h)-nK'(h).$$

The extra terms represent changes in departure capacity and in movement cost itself. At an access discontinuity the derivative is not an adequate valuation: compare the optimised restricted and available solutions. At kinks, one-sided values or finite changes are more informative than an asserted unique derivative. These qualifications are consistent with the differentiability requirements of envelope analysis.[6]

REPRODUCTION AND CHECKS

Appendix B. Reproduction and checks

The accompanying workbook contains the input register, a one-centimetre depth grid, detailed calculations, sensitivity tables and source links. Blue input values are illustrative choices; imported benchmarks are distinguished from them. Formula cells retain the calculations rather than only the reported rounded answers.

For an available route with positive payload, let $c=K/q(h)$. If $c<g$, shallow-route tonnage is $x=min\{Q,Nq(h)\}$; otherwise $x=0$. When the route is unavailable, $x=0$. Trip equivalents equal $x/q(h)$, fallback tonnage is $Q-x$, and total cost is $Kx/q(h)+g(Q-x)$, with a zero-payload guard. Repeating the calculation at $h+0.01$ produces the finite one-centimetre benefit.

The closed-form solution was checked against an independent linear-programming solver for 396 combinations of depth, alternative cost, departures and effective light draft. Freight balance and non-negative depth benefits under the model's nested-feasibility assumptions were also checked. These are computational checks, not validation against observed IJssel shipments.

An integer-departure check enumerates available whole trips and minimises $Kn+gmax\{Q-nq(h),0\}$. This yields seven trips and €39,634 at both 2.50 and 2.51 metres. It illustrates why the continuous cost curve should not be interpreted as the exact invoice saving for every single cargo batch.

The model assumes a positive payload slope and movement cost, non-negative freight and departure capacity, and an effective fallback that can carry any residual freight. Removing unlimited fallback requires an explicit feasibility or unmet-demand treatment. Multiplying results by a traffic factor is valid only if the relevant capacities scale consistently or remain non-binding; the normalisation is not permission to scale benefits without checking constraints.

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